ISSN 1984-820X
Anais do CNMAC
v.3
PUBLICAÇÃO DA SBMAC
CNMAC2010
XXXIII CONGRESSO NACIONAL DE MATEMÁTICA APLICADA E COMPUTACIONAL

Group and graph theories applied to the analysis of mechanisms and parallel robots

Roberto Simoni, Celso M. Doria,
UFSC - Departamento de Matemática,

Daniel Martins
UFSC - Departamento de Engenharia Mecânica
Campus Trindade, 88040-900, Florianópolis, SC
E-mail: roberto.emc,melkdoria,danielemc@gmail.com

Abstract: *This paper addresses applications of group and graph theories tools to simplify the analysis of kinematic chains associated with mechanisms and parallel robots. For the purpose of this analysis, a kinematic chain is described by its properties,* i.e. *degrees-of-control matrix, connectivity matrix and redundancy matrix. Kinematic chains are represented by graphs and thus, the symmetry of a kinematic chain is the same as the symmetry of its graph. The symmetry group of the graph is associated with the graph symmetry. By using the group structure induced by the symmetry we prove that degrees-of-control, connectivity and redundancy are invariants by the action of the automorphism group of the graph.*

Keywords: *kinematic chains, mechanisms, parallel robots, graph symmetry, automorphism group, actions, orbits.*

1 Introduction

Mathematical models are commonly difficult to handle in a general setting. Symmetry in mathematical models is useful to simplify the understanding of a model and to determine the patterns for which the model is appropriate. Thus, it is a common strategy to study cases of symmetry in order to learn more about a model. In our setting, the mathematical model associates a kinematic chain with a graph. Belfiore and Di Benedetto [1], Liberati and Belfiore [9] and Martins and Carboni [10] discuss how the topological structure of a kinematic chain of a parallel manipulator can be described quite extensively by degrees-of-control, connectivity and redundancy matrices. These matrices are square symmetric with dimension $n \times n$, where n is the number of links of the kinematic chain. Our aim is to apply group and graph theories to reduce the size of the degrees-of-control matrix, connectivity matrix and redundancy matrix of a kinematic chain. In this context, the graph symmetry plays an important role because it provides a group structure.

In order to achieve our aims, we investigate symmetries and invariants by the action of the automorphism group of the graph representing kinematic chains and parallel robots. Symmetries of graphs are related to automorphisms [2, 13]. By exploring these symmetries it is possible to reduce the matricial representation of properties important to the kinematic analysis of kinematic chains. It is shown that the matrix size is reduced from $n \times n$ to $o \times n$, where n is the number of links and o is the number of orbits by the action of the automorphism group of the graph. Higher graph symmetry means a smaller number of orbits o, as will be clearly shown through examples. The main result of this study was to prove that the connectivity matrix is invariant by the action of the automorphism group of the graph, the invariance is the main tool used to reduce the size of the matrices.